Abstract — Sensorless control for induction motors using Unscented Kalman Filtering is studied. The complete 6-th order dynamic model of the induction motor is analyzed and a nonlinear controller based on differential flatness theory is developed. The Unscented Kalman Filter is proposed to estimate the state vector of the nonlinear electric motor using a limited number of sensors, such as the ones measuring stator currents. Next, control of the induction motor is implemented through feedback of the estimated state vector. The efficiency of the Unscented Kalman Filter-based control scheme, is tested through simulation experiments.

I. INTRODUCTION

Induction motors are currently a main element of several industrial systems, as well as of motion transmission and transportation systems. The possibility to reduce the number of sensors involved in the control of induction motors has been a subject of systematic research during the last years [1-3]. As a result, state estimation-based control has become an active research area in the field of electric machines and power electronics. Elimination of the speed and magnetic flux sensors has the advantages of lower cost, ruggedness as well as increased reliability. Nonlinear Kalman Filtering can be used to obtain accurate estimates of the induction motor’s state vector through the processing of measurements coming from a small number of sensors, e.g. control input currents applied to stator. A well established nonlinear Kalman Filtering approach in the Extended Kalman Filter (EKF), which is based on a linearization of the nonlinear dynamics using a first order Taylor expansion [4-7]. Alternatively, the Unscented Kalman Filter (UKF) can be considered. The Unscented Kalman Filter is a derivative-free state estimation method of high accuracy. The state distribution in UKF is approximated by a Gaussian random variable [8-9]. The use of the Unscented Kalman Filter for state estimation and control of nonlinear electric motor models is a relatively new and promising topic. Indicative results on the use of the UKF for sensorless control of induction motors and fault diagnosis of electric drives can be found in [10-14].

In this paper, a sensorless control scheme for induction motors is developed consisting of (i) the Unscented Kalman Filter which provides estimates of the complete 6-th order state vector of the induction motor, after sequential processing of measurements from a limited number of sensors (such as the ones measuring stator currents), (ii) a nonlinear controller that is based on the principles of the differential flatness theory, which unlike the conventional field-oriented control approach makes no assumption about decoupling between the rotor’s magnetic flux and the rotor’s angular speed. The performance of the Unscented Kalman Filter-based sensorless control scheme is tested through simulation experiments and compared to an Extended Kalman Filter-based control loop. It is shown that the UKF-based control results also in fast and accurate trajectory tracking.

II. CONTROL OF THE FIELD-ORIENTED INDUCTION MOTOR

A. Field-oriented induction motor model

To derive the dynamic model of an induction motor the three-phase variables are first transformed to two-phase ones. This two-phase system can be described in the stator-coordinates frame $\alpha - \beta$, and the associated voltages are denoted as $v_{\alpha}$ and $v_{\beta}$, while the currents of the stator are $i_{\alpha}$ and $i_{\beta}$, and the components of the rotor’s magnetic flux are $\psi_{r\alpha}$ and $\psi_{r\beta}$. Then, the rotation angle of the rotor with respect to the stator is denoted by $\delta$, and a rotating reference frame $d-q$ on rotor, is defined [3]. The state vector of the motor is defined as $x = [\theta, \omega, \psi_{r\alpha}, \psi_{r\beta}, i_{\alpha}, i_{\beta}]$ and the dynamic model of the induction motor is written as [15-16]:

$$
\dot{x} = f(x) + g_a(x)v_{\alpha}\dot{x} + g_b(x)v_{\beta}\dot{x} + w(t) \\

z = h(x) + v(t)
$$

with the first row to describe the state equation of the motor and the second row to describe the measurement equation of the motor. The elements of the induction motor’s dynamic model are:
The use of Extended Kalman Filtering

Fig. 1. Schematic diagram the proposed flatness-based control scheme with the use of Extended Kalman Filtering

\[
f(x) = \begin{pmatrix} x_2 \\ \mu_1(x_3x_6 - x_4x_5) - T_L/\sigma \\ -\alpha_1x_3 - n_p\beta_2x_4 + \alpha_1Mx_5 \\ n_p\beta_2x_3 - \alpha_1x_4 + \alpha_1Mx_6 \\ \alpha_1\beta_1x_3 + n_p\beta_1x_2x_4 - \gamma_5 \\ -n_p\beta_1x_2x_3 + \alpha_1\beta_1x_4 - \gamma_1x_6 \end{pmatrix}
\]

\[
g_a = [0, 0, 0, 0, \frac{1}{\sigma L_r}, 0]^T, \quad g_b = [0, 0, 0, 0, 0, \frac{1}{\sigma L_s}]^T
\]

where \( J \) is the rotor’s inertia, and \( T_L \) is the external load torque. The rest of the model parameters are

\[
\sigma = 1 - M^2/L_s L_r, \quad \alpha_1 = \frac{R_d}{\sigma L_r}, \quad \beta_1 = \frac{M}{\sigma L_s L_r},
\]

\[
\gamma_1 = (\frac{M^2 R_d}{\sigma L_s L_r^2} + \frac{R_d}{\sigma L_r}), \quad \mu_1 = \frac{n_p M}{\sigma L_r}, \quad \text{where} \ L_s, L_r \text{ are the stator and rotor auto-inductances,} \ M \text{ is the mutual inductance and} \ n_p \text{ is the number of poles.}
\]

The process noise \( w(k) \) given in Eq. (1) is due to model inaccuracies associated with random variations of the model’s parameters. For example resistances, inductances and magnetic permeability of the electric motor can exhibit a stochastic variation round a nominal value. The measurement noise \( v(k) \) given in Eq. (1) is due to stochastic variations of the elements of the measuring devices. If the effects of the noise signals are not compensated by a filtering procedure, the performance of the control loop can be unsatisfactory or even the stability of the control loop can be lost. In the sensorless control scheme of the induction motor studied in this paper, the measured variables are considered to be the \( a-b \) reference frame currents of the stator.

B. Decoupling of speed-flux dynamics

The classical method for induction motors control is based on a transformation of the stator’s currents \( (i_s, \ i_q) \) and of the magnetic fluxes of the rotor \( (\psi_r, \ \psi_q) \) to the reference frame \( d-q \) which rotates together with the rotor. In the \( d-q \) frame there will be only one non-zero component of the magnetic flux \( \psi_r \), while the component of the flux along the \( q \) axis equals 0. The new control inputs of the system are considered to be \( v_{sd}, \ v_{sq} \) and are associated to the \( d-q \) frame voltages \( v_d \) and \( v_q \), respectively. The control inputs \( v_{sd}, \ v_{sq} \) are connected to \( \psi_{sd}, \ \psi_{sq} \) of Eq. (1), according to the relation

\[
\begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix} = \frac{1}{||\psi||} \begin{pmatrix} \psi_{ra} & \psi_{rb} \\ \psi_{rb} & \psi_{rb} \end{pmatrix}^{-1} \begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix}
\]

where \( \psi = \psi_{ra} \) and \( ||\psi|| = \sqrt{\psi_{ra}^2 + \psi_{rb}^2} \). Next, the following nonlinear feedback control law is defined

\[
\begin{pmatrix} v_{sd} \\ v_{sq} \end{pmatrix} = \sigma L_s \begin{pmatrix} \frac{-n_p\omega i_{sq}}{\psi_{sd}} - \frac{\alpha M i_{sd}}{\psi_{sd}} - \frac{\alpha b i_{rd}}{i_{rd}} + v_d \\ \frac{n_p\omega i_{sd}}{\psi_{sd}} + \frac{b n_p \psi_{rd}}{\psi_{rd}} + \frac{\alpha M i_{sd}}{\psi_{rd}} + v_q \end{pmatrix}
\]

Substituting Eq. (6) into Eq. (1) one obtains

\[
\begin{align*}
\frac{d}{dt} \omega &= \mu \psi_{rd} i_{sq} - \frac{T_L}{J} \\
\frac{d}{dt} i_{sq} &= -\gamma i_{sq} + v_q \\
\frac{d}{dt} \psi_{rd} &= -\alpha \psi_{rd} + \alpha M i_{sd} \\
\frac{d}{dt} i_{sd} &= -\gamma i_{sd} + v_d \\
\frac{d}{dt} \psi_{rd} &= \mu \psi_{rd} i_{sq} - \frac{T_L}{J} \\
\frac{d}{dt} i_{sq} &= -\gamma i_{sq} + v_q
\end{align*}
\]

The system of Eq. (7) to Eq. (11) consists of two linear subsystems, where the first one has as output the magnetic flux \( \psi_{rd} \) and the second has as output the rotation speed \( \omega \), i.e.

\[
\begin{align*}
\frac{d}{dt} \psi_{rd} &= -\alpha \psi_{rd} + \alpha M i_{sd} \\
\frac{d}{dt} i_{sd} &= -\gamma i_{sd} + v_d \\
\frac{d}{dt} \omega &= \mu \psi_{rd} i_{sq} - \frac{T_L}{J} \\
\frac{d}{dt} i_{sq} &= -\gamma i_{sq} + v_q
\end{align*}
\]

If \( \psi_{rd} \rightarrow \psi_{rd}^{ref} \), i.e. the transient phenomena for \( \psi_{rd} \) have been eliminated and \( \psi_{rd} \) has converged to a steady state value, then Eq. (14) is not dependent on \( \psi_{rd} \), and consequently the two subsystems described by Eq. (12)-(13) and Eq. (14)-(15) are decoupled. The subsystem that is described by Eq. (12) and Eq. (13) is linear and has as control input \( v_d \), and can be controlled using methods of DC motor control [15-17].
III. A FLATNESS-BASED CONTROL APPROACH FOR INDUCTION MOTORS

In [18] the voltage-fed induction machine was shown to be a differentially flat system. It has been proven that the angle of the rotor position (rotation angle \( \theta \)) and the angle \( \rho \) of the magnetic field (angle between flux \( \psi_r \) and \( \psi_s \)) constitute a flat output for the induction motor model [19-21]. Since all state variables of the circuits describing the induction motor dynamics can be expressed as functions of \( y = (\theta, \rho) \) and its derivatives it can be concluded that the induction motor is a differentially flat system. The equations of the induction motor in the \( d - q \) reference frame, given by Eq. (12) to Eq. (15), are now rewritten in the form of Eq. (16) to Eq. (20):

\[
\frac{d}{dt}\dot{\omega} = \mu \psi_r d i_s q - \frac{T_L}{J} \quad (16)
\]

\[
\frac{d}{dt} \psi_r d = -\alpha \psi_r d + \alpha M i_s d \quad (17)
\]

\[
\frac{d}{dt} i_s d = -\gamma i_s d + \alpha \beta \psi_r d + n_p \omega i_s q + \frac{\alpha M i_s q^2}{\psi_r d} + \frac{1}{\sigma L_s} v_s d \quad (18)
\]

\[
\frac{d}{dt} i_s q = -\gamma i_s q - \beta n_p \omega \psi_r d - n_p \omega i_s d q - \frac{\alpha M i_s d q}{\psi_r d} + \frac{1}{\sigma L_s} v_s q \quad (19)
\]

\[
\frac{d}{dt} \rho = n_p \omega + \frac{\alpha M i_s q}{\psi_r d} \quad (20)
\]

The flat outputs for the voltage-fed induction motor are the angle of the rotor \( \theta \) and variable \( \rho \), where \( \rho \) has been defined as the rotor flux angle. According to [21], if the stator current dynamics are much faster than the speed and flux dynamics a faster inner current control loop can be designed using only Eq. (18) and Eq. (19) and assuming the speed and flux as constants. For the outer speed and flux control design, the stator currents are treated as new control inputs and the system behavior is described by Eq. (16), Eq. (17) and Eq. (20). This system of lower order is also flat with \( \psi_r d \) and \( \theta \) as flat outputs. It can be shown that all state variables of the induction motor can be written as functions of the flat outputs and their derivatives. Moreover, from Eq. (18) and Eq. (19) a controller that satisfies the flatness properties (and which can be also expressed as a function of the flat outputs and their derivatives) is:

\[
v_s d = \sigma L_s \left( \frac{d i_s^*}{dt} + \gamma i_s^* + \beta \psi_r d - n_p \omega i_s q - \frac{\alpha M i_s q^2}{\psi_r d} + v_s d \right) \quad (21)
\]

\[
v_s q = \sigma L_s \left( \frac{d i_s^*}{dt} + \gamma i_s^* + \beta n_p \omega \psi_r d + n_p \omega i_s d q + \frac{\alpha M i_s d q}{\psi_r d} + v_s q \right) \quad (22)
\]

where \( i_s^* \) and \( i_s^* d \) denote current setpoints. Substituting Eq. (21) and Eq. (22) into Eq. (18) and Eq. (19) one obtains the dynamics of the current tracking errors.

\[
\frac{d}{dt} i_s d = -\gamma i_s d + v_s d \quad (23)
\]

\[
\frac{d}{dt} i_s q = -\gamma i_s q + v_s q \quad (24)
\]

where \( \Delta i_s d = (i_s d - i_s^*) \). For the decoupled system of Eq. (23) and Eq. (24) one can apply state feedback control. For example a suitable state feedback controller would be

\[
v_s d = -\gamma_1 \Delta i_s d \quad (25)
\]

\[
v_s q = -\gamma_2 \Delta i_s q \quad (26)
\]

Tracking of the reference setpoint can be also succeeded for the rotor’s speed and flux through the application of the control law of Eq. (21) and Eq. (22) to Eq. (16) and Eq. (20). The control inputs are chosen as

\[
i_s d = \frac{1}{\alpha M} \left( \frac{d \psi_r^*}{dt} + \alpha \psi_r^* + i_d \right) \quad (27)
\]

\[
i_s q = \frac{1}{\mu \psi_r d} \left( \frac{d \omega}{dt} + i_q \right) \quad (28)
\]

Denoting \( \psi_r d = \psi_r d - \psi_r^* \) and \( \Delta \omega = \omega - \omega^* \) the tracking error dynamics are given by

\[
\frac{d}{dt} \Delta \psi_r d = -\alpha \Delta \psi_r d + i_d \quad (29)
\]

\[
\frac{d}{dt} \Delta \omega = -\frac{T}{J} + i_q \quad (30)
\]

The convergence of the tracking error to zero can be assured through the application of the following feedback control laws:

\[
i_d = -\alpha_1 \Delta \psi_r d \quad (31)
\]

\[
i_q = \frac{T}{J} - \alpha_2 \Delta \omega \quad (32)
\]

IV. EXTENDED KALMAN FILTERING FOR INDUCTION MOTOR CONTROL

For sensorless control of nonlinear systems, such as induction motors, the EKF is an established state estimation approach [2]. Observability issues of the induction motor have been studied in [22]. The following nonlinear state model is considered [3-7]:

\[
x(k + 1) = \phi(x(k)) + L(k)u(k) + w(k)
\]

\[
z(k) = \gamma(x(k)) + v(k)
\]

where \( x \in \mathbb{R}^{n \times 1} \) is the system’s state vector and \( z \in \mathbb{R}^{p \times 1} \) is the system’s output, while \( u(k) \) and \( v(k) \) are uncorrelated, zero-mean, Gaussian noise processes with covariance matrices \( Q(k) \) and \( R(k) \) respectively. The operators \( \phi(x) \) and \( \gamma(x) \) are vectors defined as \( \phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_m(x)]^T \) and \( \gamma(x) = [\gamma_1(x), \gamma_2(x), \ldots, \gamma_p(x)]^T \). Following a linearization procedure, \( \phi \) is expanded into Taylor series about \( \dot{x} \) and the Jacobian \( J_\phi(x) \) is calculated at \( \dot{x} \). Similarly, following a linearization procedure, \( \gamma \) is expanded into Taylor series about
\[ \hat{x}_k = f(x_{k-1}, u_{k-1}) + q_{k-1} \]
\[ z_k = h(x_k) + r_k \]

where \( x_k \in \mathbb{R}^{n \times 1} \) is the state, \( z_k \in \mathbb{R}^{p \times 1} \) is the measurement, \( q_{k-1} \in \mathbb{R}^{p \times 1} \) is a Gaussian process noise \( q_{k-1} \sim N(0, Q_{k-1}) \), and \( r_k \in \mathbb{R}^{p \times 1} \) is a Gaussian measurement noise \( r_k \sim N(0, R_k) \). The mean and covariance of the initial state \( x_0 \) are \( m_0 \) and \( P_0 \), respectively. Some basic operations performed in the UKF algorithm (Unscented Transform) are as follows:

1) Denoting the current state mean as \( \hat{x} \), a set of \( 2n + 1 \) sigma points are taken from the columns of the \( n \times n \) matrix \( \sqrt{(n + \lambda)P_{xx}} \) as follows: \( x_0^i = \hat{x}, x_i^i = \hat{x} + \sqrt{(n + \lambda)P_{xx}}, x_i^i = \hat{x} - \sqrt{(n + \lambda)P_{xx}} \) and the associate weights are computed: \( W_0 = \frac{1}{n + \lambda}, W_i = \frac{1}{2(n + \lambda)}, i = 1, \cdots, 2n \). Matrix \( P_{xx} \) is the covariance matrix of the state \( x \). The parameter \( \lambda \) is a scaling parameter.

2) Transform each of the one of the sigma points which are defined as \( z_i = h(x_i), i = 0, \cdots, 2n \).

3) Mean and covariance estimates for \( z \) can be computed as \( \hat{z} = \sum_{i=0}^{2n} W_i z_i, P_{zz} = \sum_{i=0}^{2n} W_i (z_i - \hat{z}) (z_i - \hat{z})^T \).

4) The cross-covariance of \( x \) and \( z \) is estimated as \( P_{xz} = \sum_{i=0}^{2n} W_i (x_i - \hat{x}) (z_i - \hat{z})^T \).

The matrix square root of positive definite matrix \( P_{xx} \) means a matrix \( A = \sqrt{P_{xx}} \) such that \( P_{xx} = AA^T \) and a possible way for calculation is SVD [7]. The UKF also consists of prediction stage (time update) and correction stage (measurement update) [25].

**Time update:**
Compute the predicted state mean \( \hat{x}_{k-} \) and the predicted covariance \( P_{xxk-} \) as

\[ \hat{x}_{k-}, P_{xxk-} = UT(f_d, \hat{x}_{k-1}, P_{xxk-1}) \]
\[ P_{xxk-} = P_{xxk-1} + Q_{k-1} \] (37)

**Measurement update:**
Obtain the new output measurement \( z_k \) and compute the predicted mean \( \hat{z}_k \) and covariance of the measurement \( P_{zzk} \), and the cross covariance of the state and measurement \( P_{xz}\)

\[ \hat{z}_k, P_{zzk}, P_{xz} = UT(h_d, \hat{x}_{k-}, P_{xxk-}) \]
\[ P_{zzk} = P_{zzk} + R_k \] (38)

Then compute the filter gain \( K_k \), the state mean \( \hat{x}_k \) and the covariance \( P_{xxk}, \) conditional to the measurement \( y_k \)

\[ K_k = P_{xz} P_{zzk}^{-1} \]
\[ \hat{x}_k = \hat{x}_{k-} + K_k (z_k - \hat{z}_k) \]
\[ P_{xxk} = P_{xxk-} - K_k P_{zzk} K_k^T \] (39)

The filter starts from the initial mean \( m_0 \) and covariance \( P_{xx0} \).

**VI. SIMULATION RESULTS**

The flatness-based control method for the induction motor that was presented in Section III requires knowledge of the electric motor’s state vector \( x = [\theta, \omega, i_s, i_q, i_s, \rho] \). It will be shown that it is possible to implement state estimation for the electric motor using measurements only of the stator currents \( i_s, i_q \) and \( i_s \). A nonlinear Kalman Filter, such as the UKF or the EKF, can give estimates of the non-measured state vector elements, i.e., of the rotor’s angle \( \theta \), of the rotation speed \( \omega \), of the magnetic flux \( \psi_r \), and of the angle \( \rho \) between the flux vectors \( \psi_r \) and \( \psi_r \). Using currents \( i_s, i_q \) and \( i_s \) and the estimate of angle \( \rho \), the input measurements \( i_s, i_q \) and \( i_s \) can be provided to the nonlinear Kalman Filters. Thus one has

\[ \begin{pmatrix} i_s \\ i_q \end{pmatrix} = \begin{pmatrix} \cos(\hat{\rho}) & \sin(\hat{\rho}) \\ -\sin(\hat{\rho}) & \cos(\hat{\rho}) \end{pmatrix} \begin{pmatrix} i_s \\ i_q \end{pmatrix} \] (40)

The performance of the proposed sensorless control scheme, which uses the UKF for estimation of the non-measurable parameters of the motor’s state vector, is depicted in Fig. 2 to Fig. 5 (tracking of a sinusoidal setpoint) and in Fig. 6 to Fig. 9 (tracking of a seesaw setpoint). The red line denotes the reference setpoint, the blue line denotes the real value of the state variable and the green line stands for the value of the estimated state variable. Comparison to a sensorless control loop that is based on the EKF is also provided. To implement the EKF the Jacobian matrices \( J_\phi \) and \( J_\gamma \), described in Section IV, have to be calculated. It is noted that if a Jacobian matrix \( J_\phi \) associated to the drift term of the system’s dynamics, is computed using the system’s continuous-time description of Eq. (1), then in the EKF recursion of Eq. (34) and (35) it should be substituted by \( I + T_s J_\phi \) where \( T_s \) is the sampling period and \( I \in \mathbb{R}^{n \times n} \) is the identity matrix. From the simulation experiments it can be observed that the UKF-based control results in fast and accurate trajectory tracking. The performance of the UKF-based control loop, when considering as measured variables only the stator currents, was comparable to the one of the EKF-based control loop. Methods to further

\[ \hat{x}(k) = \hat{x}_0 + \sum_{i=0}^{2n} W_i \hat{x}_i \]
\[ P(k) = P(0) + \sum_{i=0}^{2n} W_i (\hat{x}_i - \hat{x})^T (\hat{x}_i - \hat{x}) \]
\[ \hat{x}_k = \hat{x}_{k-} + K_k (z_k - \hat{z}_k) \]

\[ K_k = P_{xz} P_{zzk}^{-1} \]
\[ \hat{x}_k = \hat{x}_{k-} + K_k (z_k - \hat{z}_k) \]
\[ P_{xxk} = P_{xxk-} - K_k P_{zzk} K_k^T \] (39)
enhance the robustness of the nonlinear filtering-based control loops have been discussed in [27-28].

![Graph](image)

Fig. 2. Angle $\theta$ of the induction motor (blue line) in sensorless control when tracking a sinusoidal setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

![Graph](image)

Fig. 3. Angular velocity $\omega$ of the induction motor (blue line) in sensorless control when tracking a sinusoidal setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

![Graph](image)

Fig. 4. Control input current $i_d$ of the induction motor (blue line) in sensorless control when tracking a sinusoidal setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

![Graph](image)

Fig. 5. Control input current $i_q$ of the induction motor (blue line) in sensorless control when tracking a sinusoidal setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

![Graph](image)

Fig. 6. Angle $\theta$ of the induction motor in sensorless control (blue line) when tracking a seesaw setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

![Graph](image)

Fig. 7. Angular velocity $\omega$ of the induction motor (blue line) in sensorless control when tracking a seesaw setpoint (red line) and state estimation is performed with (a) the Extended Kalman Filter (b) Unscented Kalman Filter.

VII. CONCLUSIONS

A sensorless control scheme for induction motors has been proposed consisting of (i) the Unscented Kalman Filter which provides estimates of the complete 6-th order state vector of the induction motor after sequential processing of measurements from a small number of sensors (such as the ones measuring stator currents), (ii) a nonlinear controller that is based on the principles of the differential flatness theory, which unlike the conventional field-oriented control approach makes
no assumption about decoupling between the rotor's magnetic flux and the rotor's angular speed. Simulation tests were used to evaluate the UKF-based control loop. It was shown that the UKF is a reliable and computationally efficient approach to state estimation-based control. It was demonstrated that the UKF results in accurate trajectory tracking. Moreover, it was shown that the fast recursion of the UKF makes it suitable for use in real-time control problems.

REFERENCES